Noţiuni de Biostatistică: statistică superioară

Modelarea matematică. Noţiuni generale

Aplicații ale diferitelor capitole din matematică în diverse discipline medicale

X = aplicații majore
o = alte aplicații

	Informatica medicală	Biofizică	Fiziologie	Imunologie	Biochimie	Genetică	Anatomie	Farmacologie	Medicină nucleară	Explorări funcționale	Ecologie	Microbiologie	Igienă	Histologie, citologie	Patologie	Radiologie, imagistică	Medicină internă	Chirurgie
Statistica (elem)	X	X	X	X	X	X	X	X	X	X	X	X	X	o	o	X	X	X
Statistica (dezv)	X	X	X	X	X	X	o	X	X	X	X	X	X	o		o	X	o
Algebra (elem)	X	X	X	o	X	o	o	X	o	o	X	o	X	o				
Calcul diferențial şi Ecuații diferențiale	X	X	X	X	X	o		X	X	X	X	o	o	o				
Calcul integral şi Ecuații integrale	X	X	o	o	X			X	X	X	X	o						
Geometrie şi Trigonometrie	o	X	X	o	o	o	X		X	o	o			X	X	X		
Topologie şi Geometrie analitică	X	X	X	X	o	o	X		X	o	o			X	X	X		
Metode numerice	X	X	X	X	X	X	X	X	X	X	X	X	o	o		X	o	
Funcții complexe şi Analiza Fourier	X	X	X	X	X		o	o	X	X						X		
Analiza combinatorie	X	X	X	X	X	X	X	X	o		X	X	o	o	o		o	
Teoria informației	X	X	X	X	X	X	o				X			o			X	o
Logica matematică	X	o	o	X	o	X	o					o					X	o
Probabilități	X	X	o	X	o	X	o	X	o		X	o	o	o			o	
Mulțimi fuzzy	X	X	X	X	o	X	X	X	X		X	o				o	o	
Teoria haosului	X	X	X	X	o	X	o	o		o	X	o	X	o			o	
Fractali	X	X	X	X	o	X	X				X	o	o	X	X	o		
Teoria bifurcației	X	X	X	X	o	X	X	o		o	X	o	X	X		o	o	
Teoria catastrofelor	X	X	X	X	o	X	o	o			X	o	X	o			o	

NOTIUNI GENERALE

- **DEFINITII:**
  - **MODELARE - metoda de studiu**
  - **MODEL - reprezentare simplificata**
    - **modele fizice - machete**
    - **modele teoretice**
  - **SIMULARE - procedeu de urmarire a comportarii unui model**

APLICATII

- PROIECTAREA EXPERIENTELOR
- VERIFICAREA UNOR IPOTEZE
- DETERMINAREA UNOR PARAMETRI
- PROGNOZA
- APROFUNDAREA FENOMENULUI
- APLICATII DIDACTICE

CLASIFICAREA MODELELOR

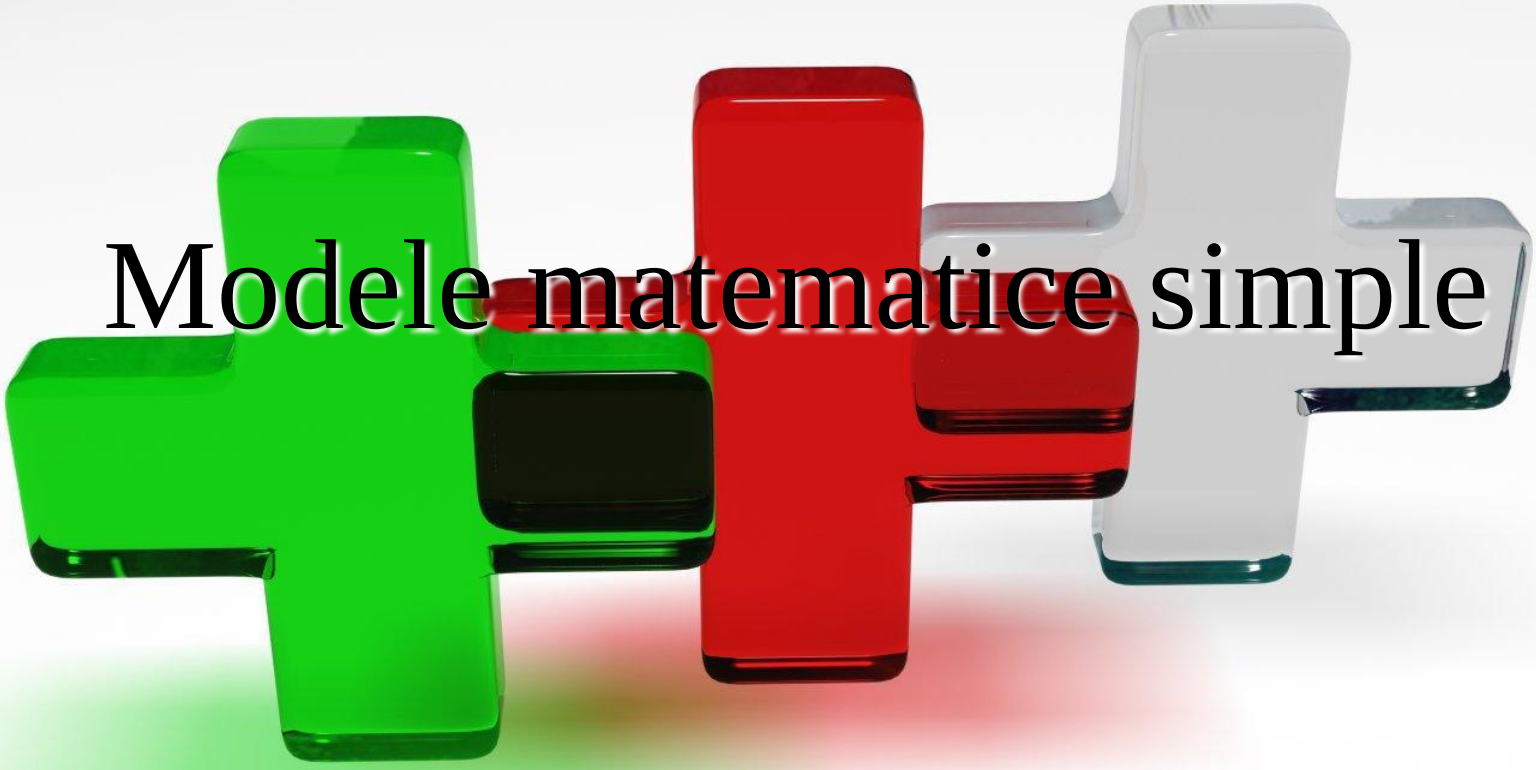
- DUPA STRUCURA SISTEMULUI / PROCESULUI
  - M. CONTINUE
  - M. DISCRETE
- DUPA RELATIA INTRARE / IESIRE
  - M. DETERMINISTE
  - M. STOCHASTICE
- DUPA CONSTRUCTIE
  - M. EMPIRICE
  - M. TEORETICE

- DUPA ALGORITMUL FOLOSIT
  - M. CU ECUATII ANALITICE
  - M. CU ECUATII DIFERENTIALE
  - M. PROBABILISTICE (MONTE-CARLO)
  - M. FUZZY
- DUPA ASPECTUL MODELAT
  - M. DE STRUCTURA
  - M. DE FUNCTIE
- DUPA DESFASURAREA TEMPORALA
  - M. STATICE
  - M. DINAMICE

CONSTRUCTIA UNUI MODEL

- STABILIREA (DELIMITAREA) SISTEMULUI SI A PROCESULUI MODELAT
- ALEGEREA VARIABILELOR (dependente / ind.)
- STABILIREA RELATIILOR DINTRE VARIABILE
- STABILIREA MARIMILOR DE INTRARE SI IESIRE
- VALORI INITIALE, PARAMETRI SI CONSTANTE
- MODUL DE PREZENTARE A REZULTATELOR (tabel, grafic evolutie, diagrama faze, animatie etc)
- STABILIREA MODULUI DE VALIDARE

Modele matematice simple



MODELE DETERMINISTE SIMPLE

CRESTEREA EXPONENTIALA CONTINUA

a) ELABORAREA MODELULUI

initial: 0 . . . . . N_0 (S_0)

mom. t . . . . . N (S)

timp t + dt . . . . N + dN

---

dN ~ N si **dN ~ dt** => **dN = r . N . dt**

- **b) FORMELE MODELULUI**

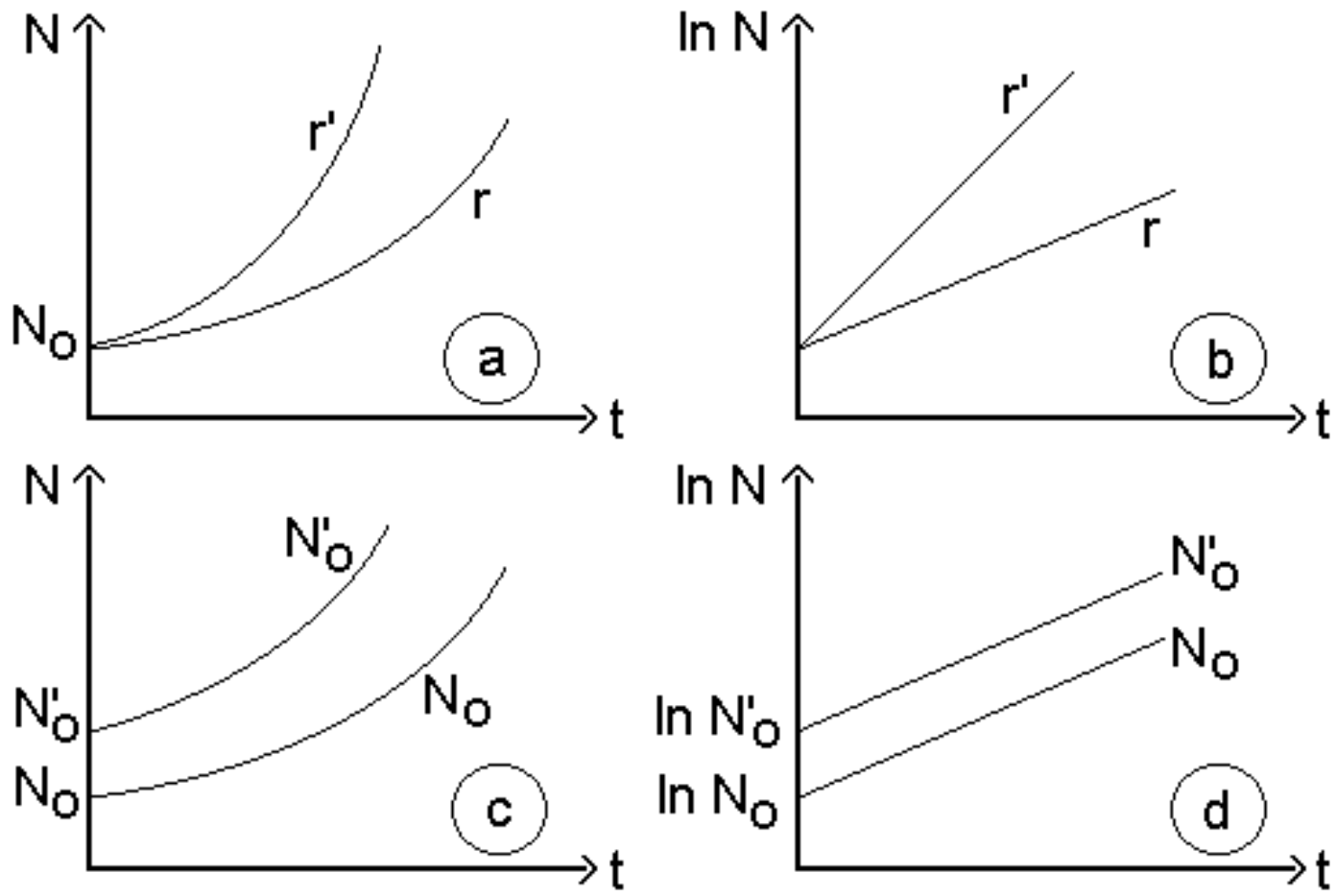
  – **FORMA DIFERENTIALA**

$$dN / dt = r \cdot N$$

  – **FORMA ANALITICA**

$$N = N_0 \cdot e^{r t}$$

- **c) REPREZENTARI GRAFICE**


N
r'
r
N_o
a
t
ln N
r'
r
b
t
N
N'_o
N_o
N'_o
N_o
c
t
ln N
N'_o
N_o
ln N'_o
ln N_o
d
t

- **d) SCADEREA EXPONENTIALA**
- **e) CRESTEREA EXPONENTIALA DISCRETA**
  - **NU APARE TIMPUL**
  - **APAR INDICI REPREZENTAND ‘GENERATII’**

$$N_{i+1} = R \cdot N_i$$

$R > 1$ = CRESTERE

$R < 1$ = SCADERE

CRESTERA LIMITATA

- a) MODELUL MONOMOLECULAR
  - ELABORAREA MODELULUI

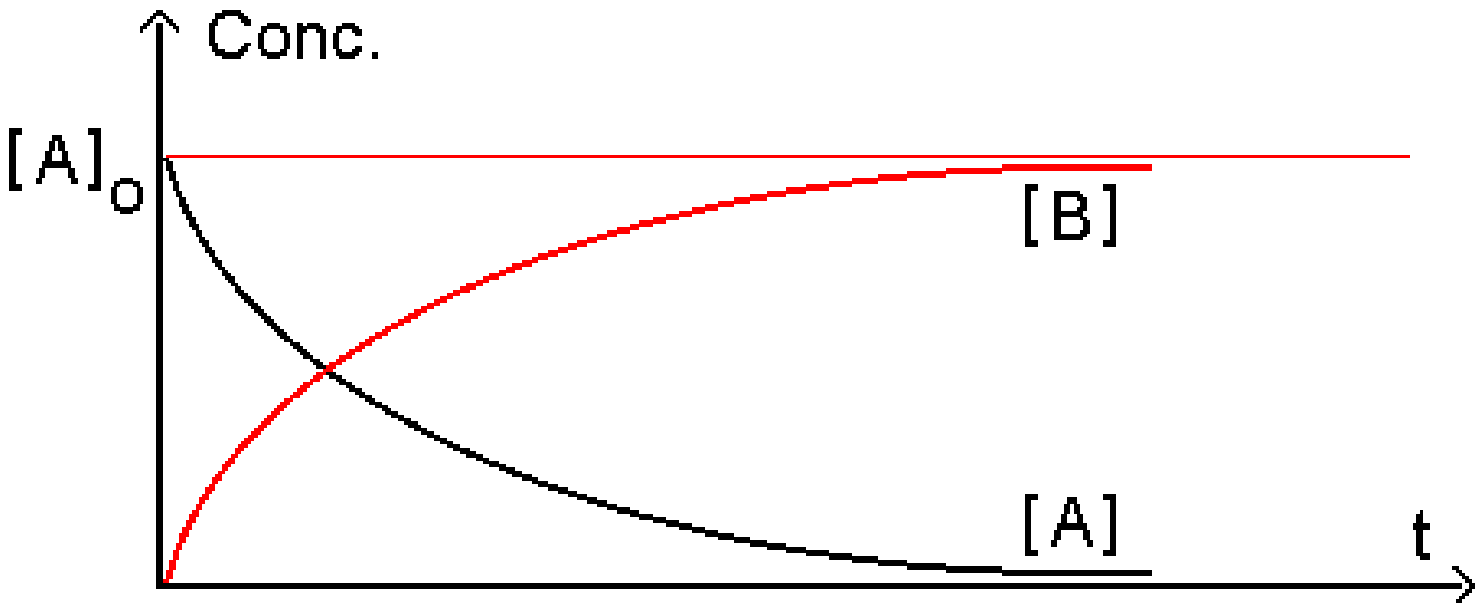
$$A \longrightarrow B \quad \text{(rata 'k')}$$

  - FORMA DIFERENTIALA

$$d[A] / dt = - k [A]$$

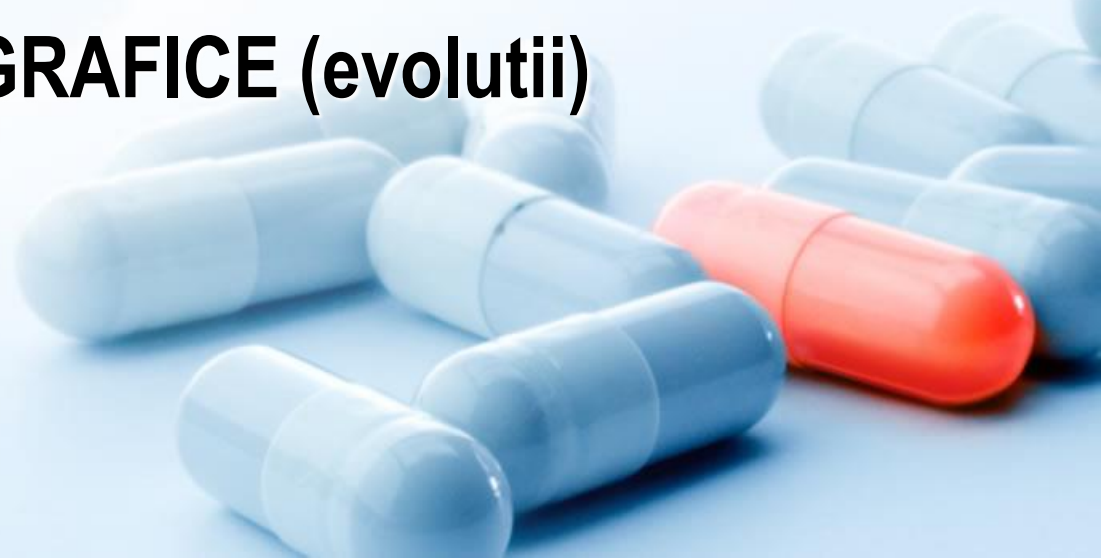
$$d[B] / dt = + k [A]$$

  - REPREZENTARE GRAFICA


Conc.
[A]o
[B]
[A]
t

b) MODELAREA PROCESELOR BIOCHIMICE (SI MODELE COMPARTIMENTATE)

- ELABORAREA MODELULUI
- FORMA DIFERENTIALA
- REPREZENTARI GRAFICE (evolutii)



Modelarea reactiilor biochimice

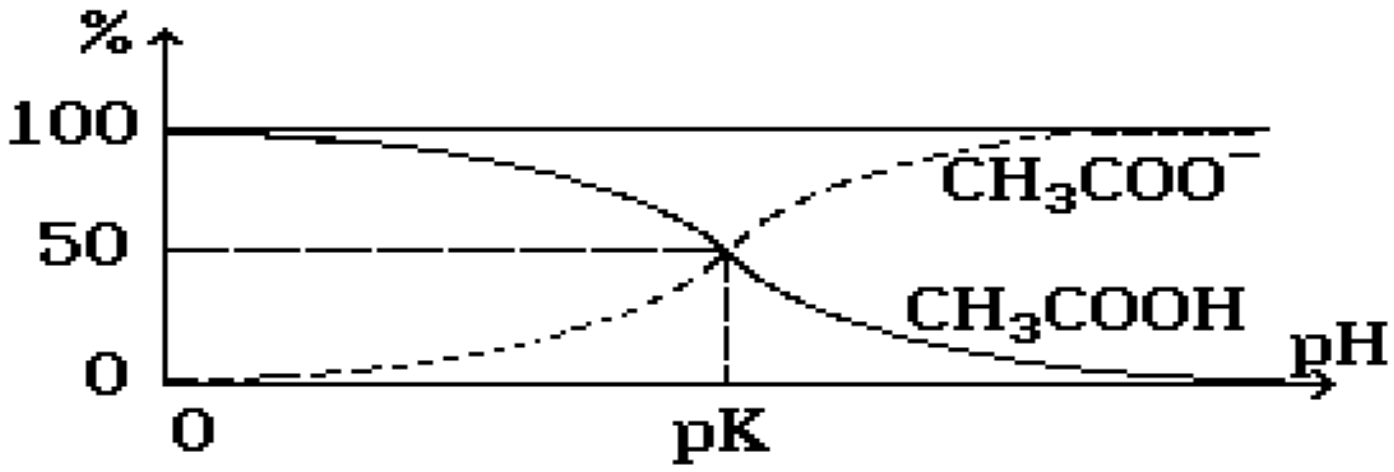
$$A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} B + C$$

$$\frac{d[A]}{dt} = -k_1[A] + k_2[B]\,[C]$$

$$\frac{d[B]}{dt} = \frac{d[C]}{dt} = k_1[A] - k_2[B]\,[C]$$

$$\frac{[B][C]}{[A]} = \frac{k_1}{k_2} = K \qquad (d[A]/dt = 0)$$

Simularea disocierii acidului acetic



MODELUL LOGISTIC CONTINUU

- RATA DE CRESTERE SCADE LA APROPIEREA DE ECHILIBRU (K):

$$r' = r\,(1 - N / K)$$

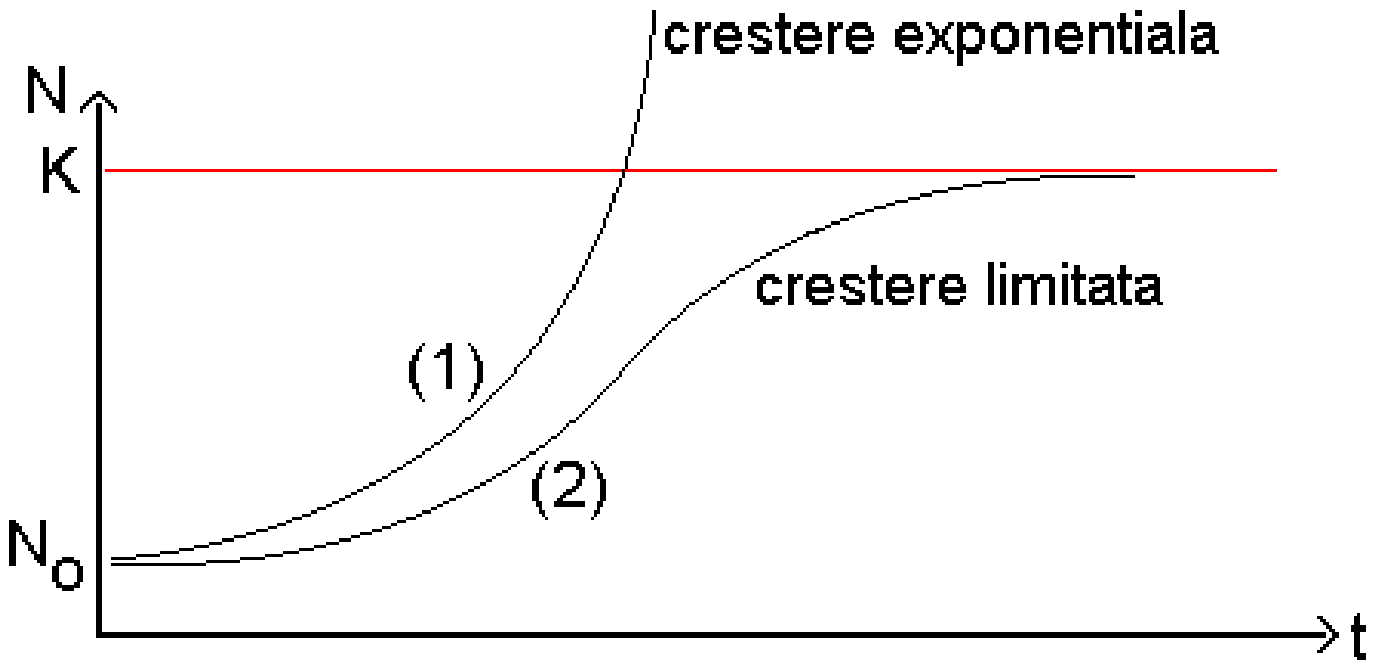
- FORMA DIFERENTIALA

$$dN / dt = r\,(1 - N / K)\,.\,N$$

- FORMA ANALITICA

$$N = K / (1 + (K / N_0 - 1)\,.\,e^{-rt}$$

- REPREZENTARE GRAFICA (sigmoida)



MODELUL LOGISTIC DISCRET

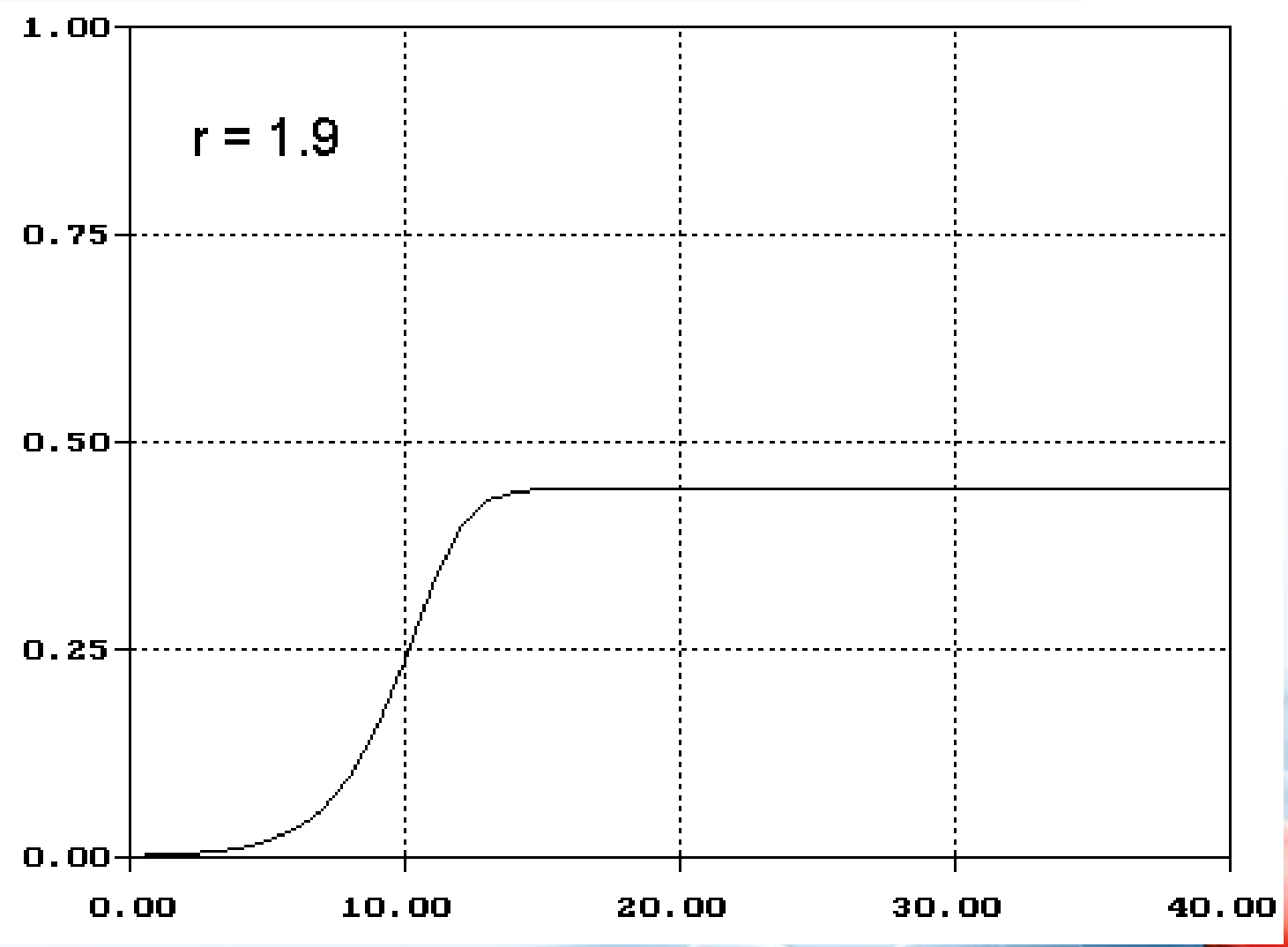
- **ELABORAREA MODELULUI**

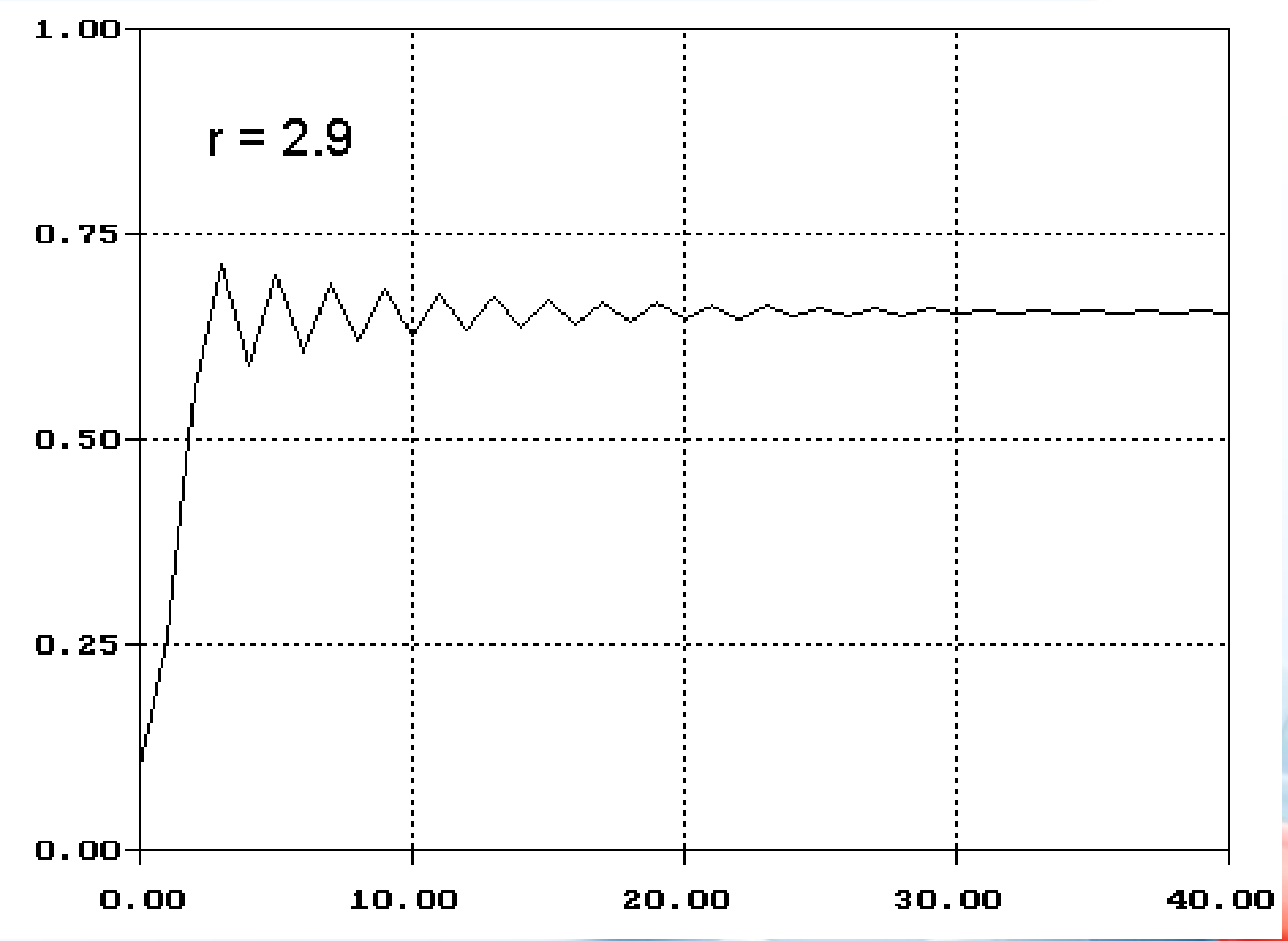
$$N_{i+1} = R \cdot N_i \cdot (1 - N_i / K)$$

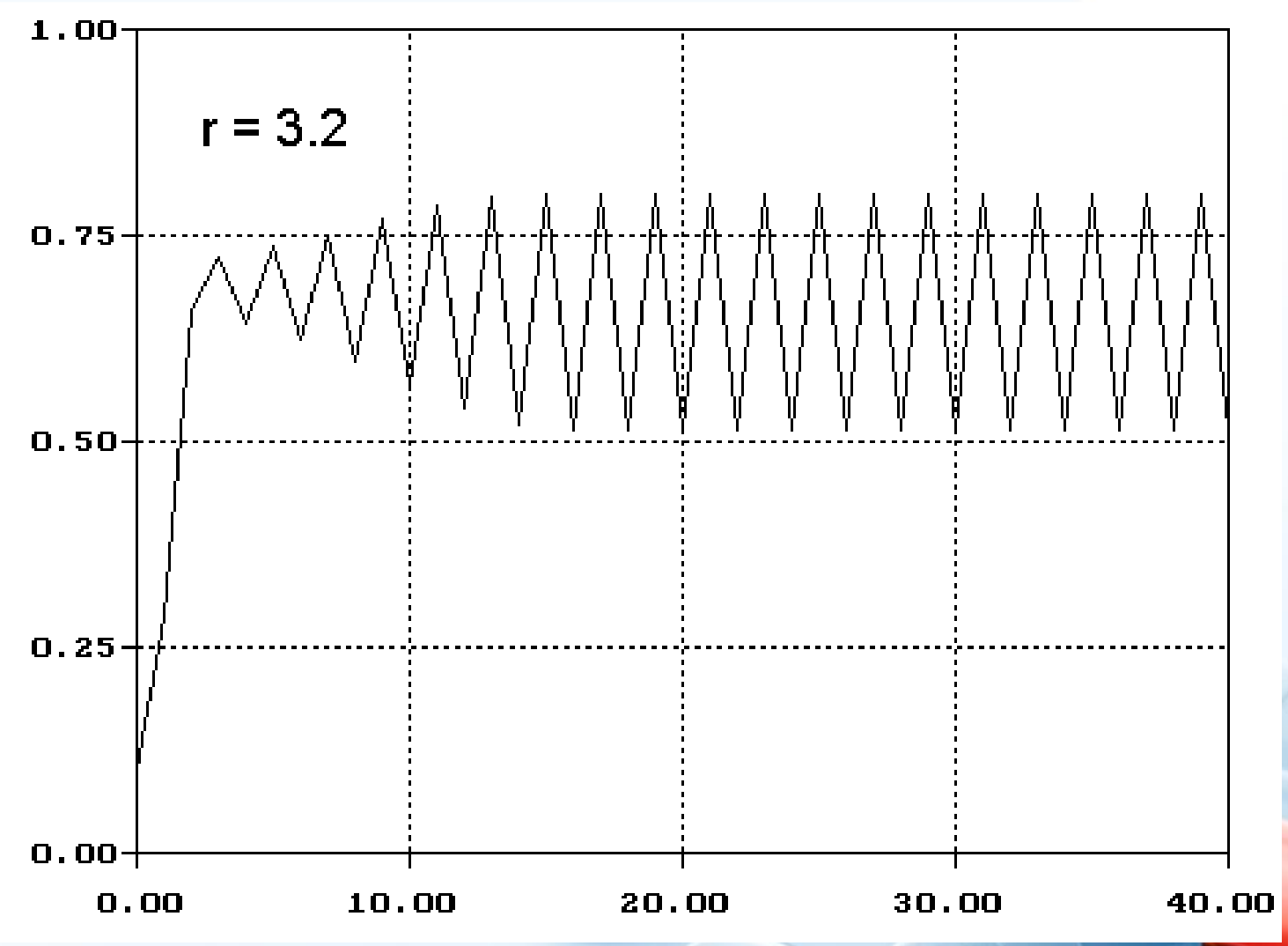
notam $X_i = N_i / K$

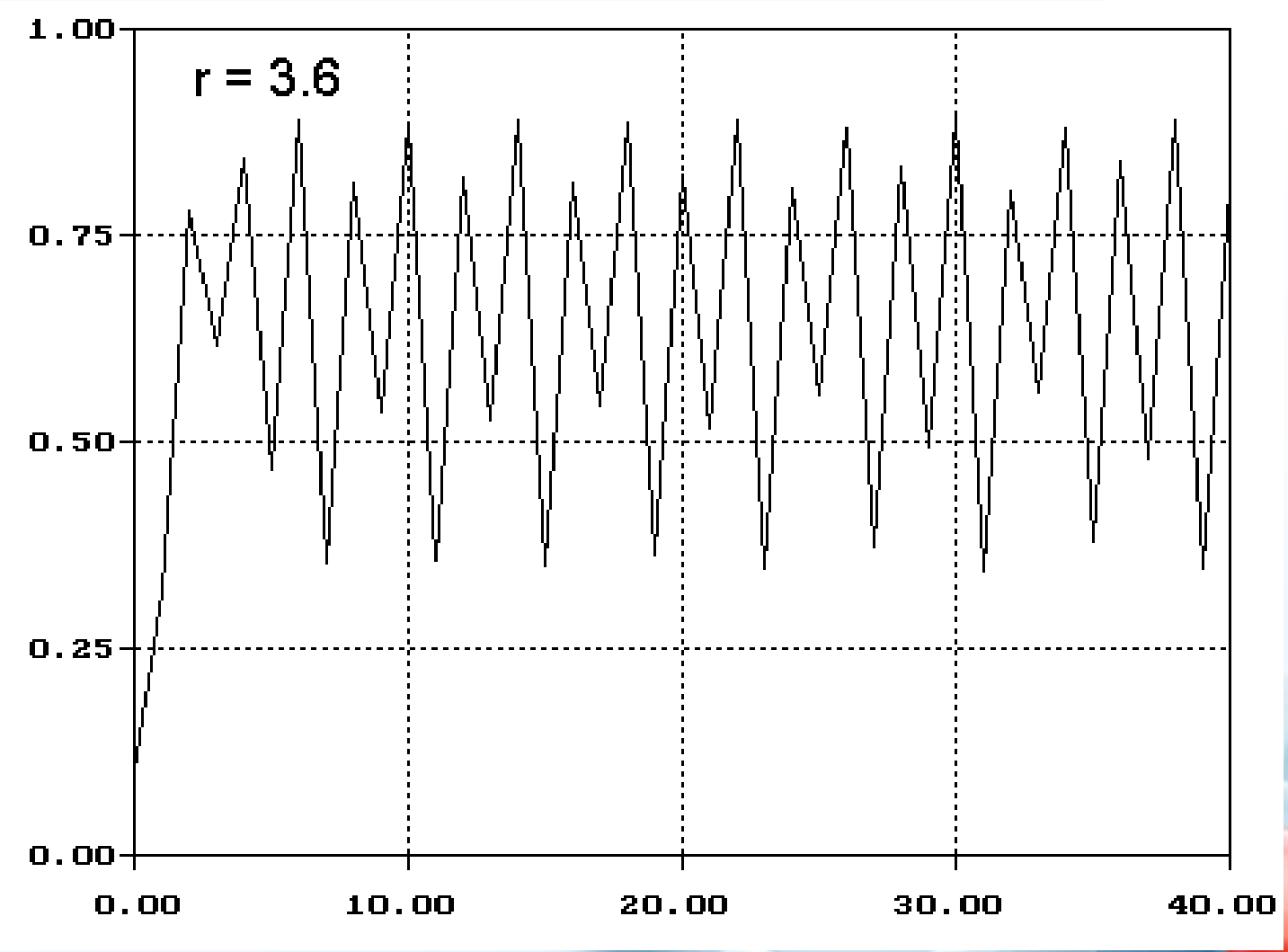
$$\boxed{X_{i+1} = R \cdot X_i (1 - X_i)}$$

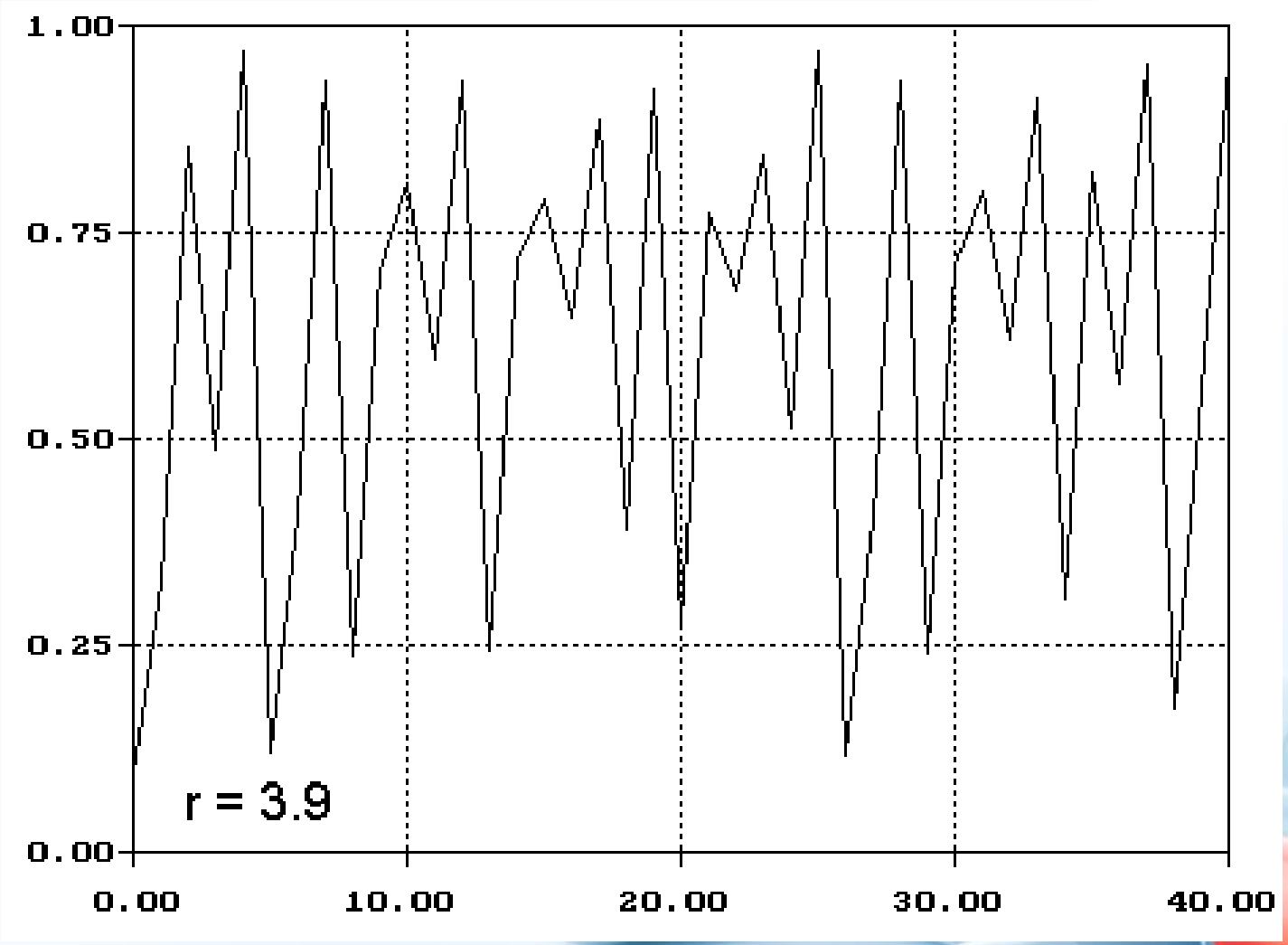
- **COMPORTAREA MODELULUI**
- **TEORIA HAOSULUI**

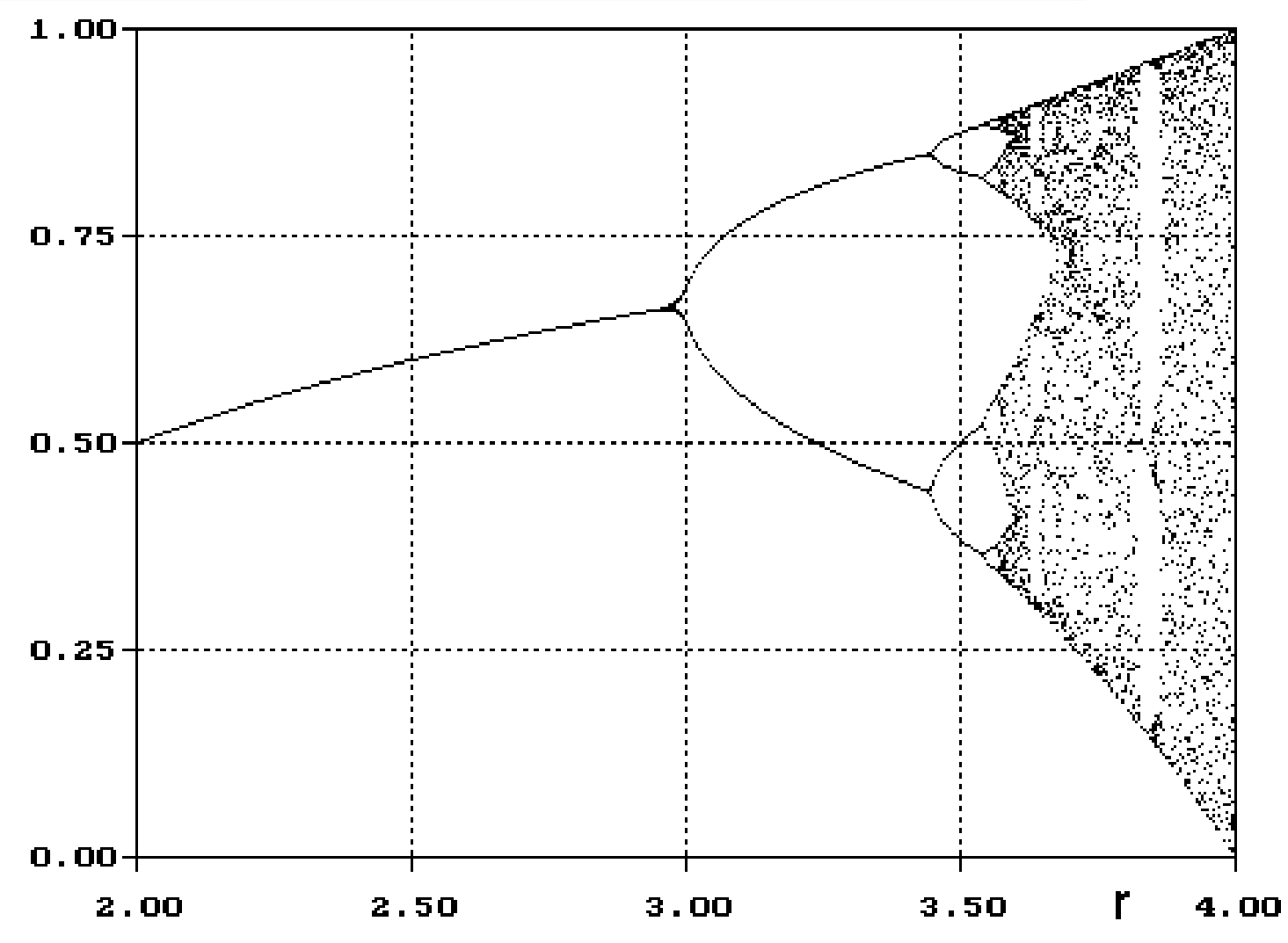

r = 1.9
1.00
0.75
0.50
0.25
0.00
0.00
10.00
20.00
30.00
40.00


r = 2.9
1.00
0.75
0.50
0.25
0.00
0.00
10.00
20.00
30.00
40.00

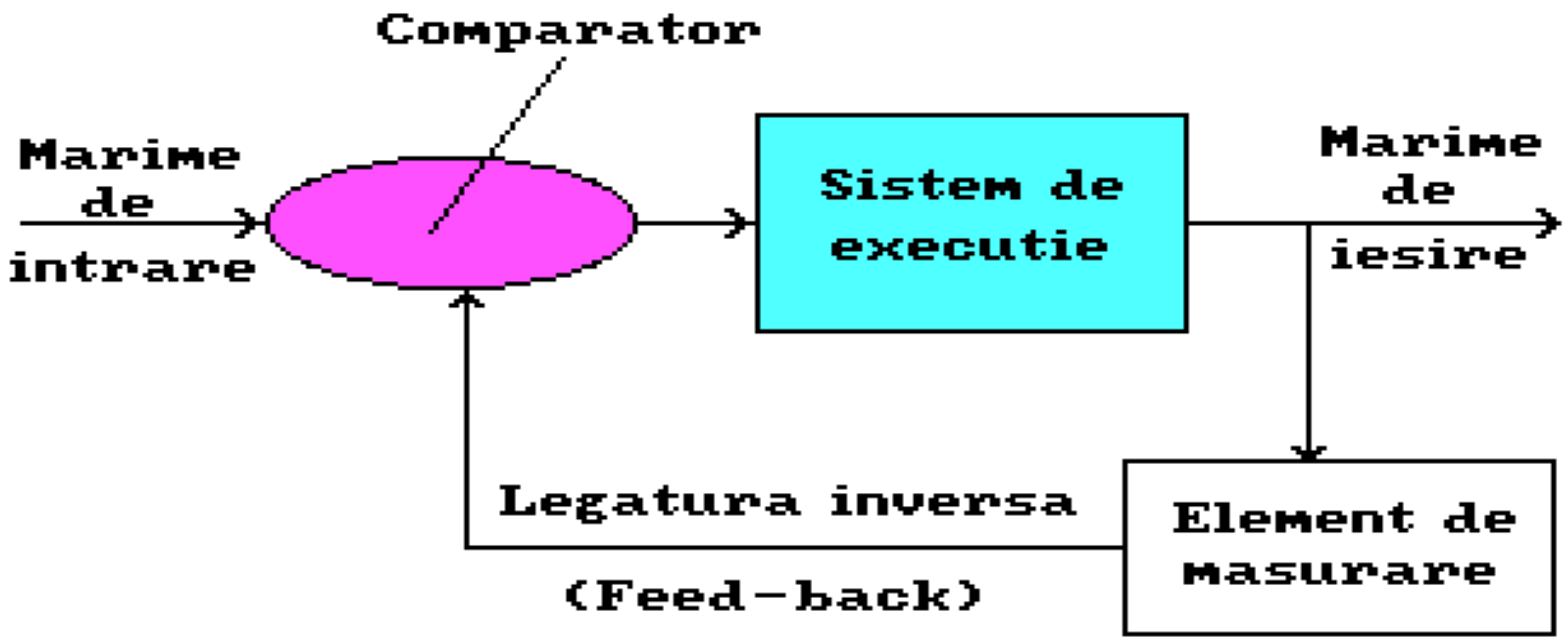

r = 3.2
1.00
0.75
0.50
0.25
0.00
0.00
10.00
20.00
30.00
40.00


r = 3.6
1.00
0.75
0.50
0.25
0.00
0.00
10.00
20.00
30.00
40.00


1.00
0.75
0.50
0.25
0.00
0.00
10.00
20.00
30.00
40.00
r = 3.9

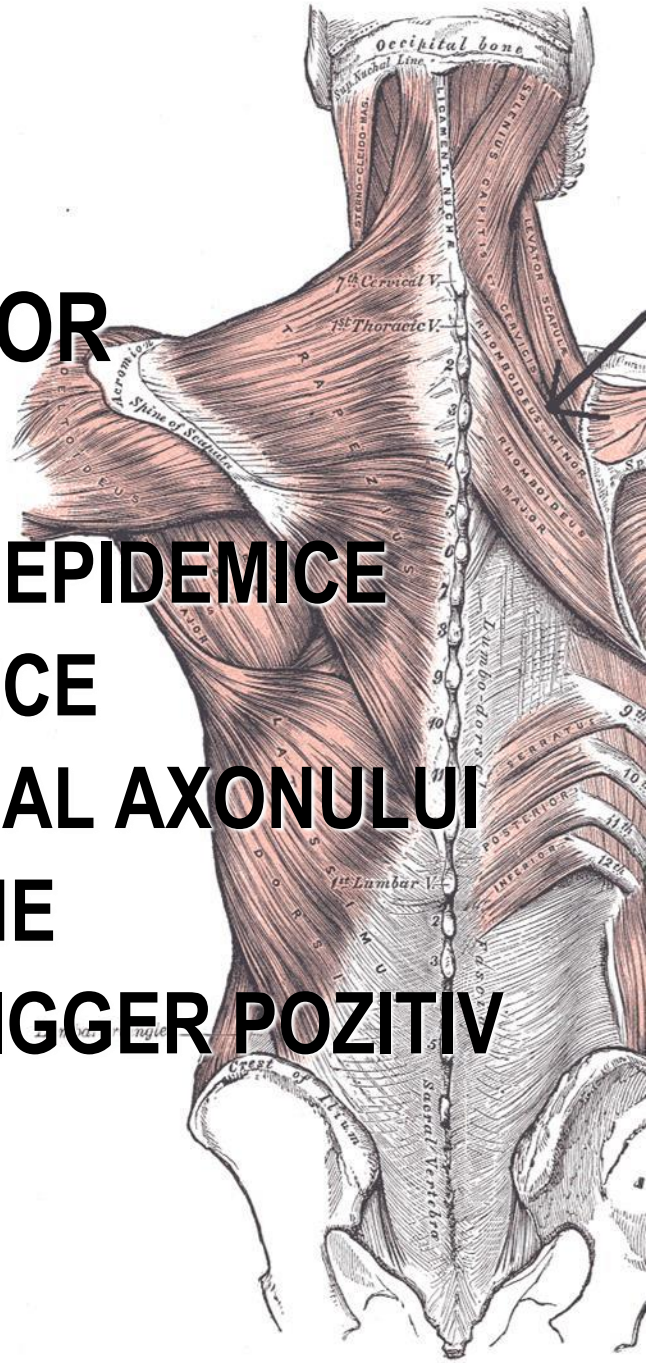

1.00
0.75
0.50
0.25
0.00
2.00
2.50
3.00
3.50
r
4.00

Schema unui sistem cu reglare automata



Modele stochastice, aplicaţii în genetică

- MODELE STOCHASTICE
- MODELAREA STRUCTURILOR
- MODELE DEZVOLTATE
  - M. PRADATOR – PRADA, M. EPIDEMICE
  - M. PROCESELOR FIZIOLOGICE
  - POTENTIALUL DE ACTIUNE AL AXONULUI
  - MODELAREA RETELEI IMUNE
  - REGLAJUL GENETIC CU TRIGGER POZITIV
- RETELE NEURONALE



MODELE STOCHASTICE

MODELAREA DEZINTEGRARII RADIOACTIVE

Notatii:

N_o = nr. initial de nuclee

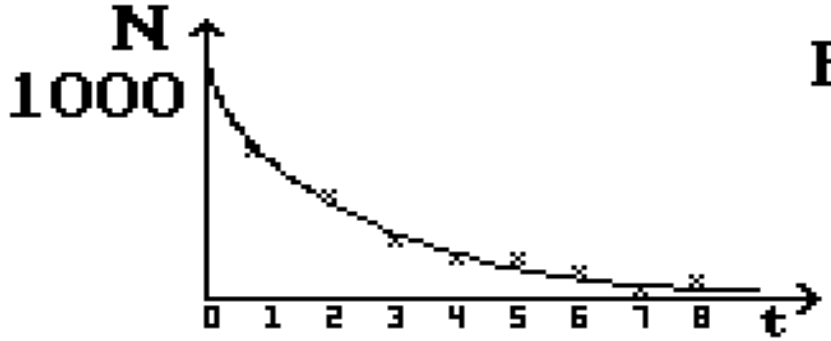
N = nr. nuclee la momentul t

dN = nr. nuclee dezint. in unitate de timp

λ = probabilitate de dezintegrare a unui nucleu in timp

Algoritm:

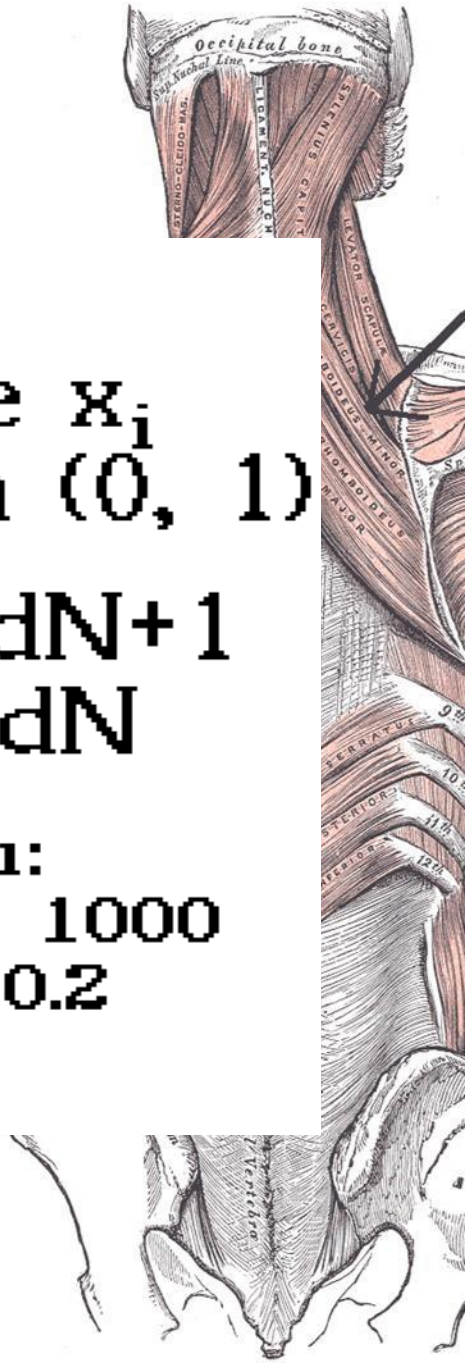
- generam N nr. aleatoare x_i cu distributie uniforma in (0, 1)
- daca $x_i > \lambda$ atunci $dN \leftarrow dN+1$
- in ciclul urmator $N \leftarrow N-dN$



Exemplu:

$N_0 = 1000$

$\lambda = 0.2$



Modele genetice

a) Legile lui Mendel
b) Principiul lui Hardy-Weinberg
c) Simularea monohibridismului
d) Simularea dihibridismului

genotip parinti AA aa

gameti A a

g. prima gener. Aa Aa

gameti A a A a

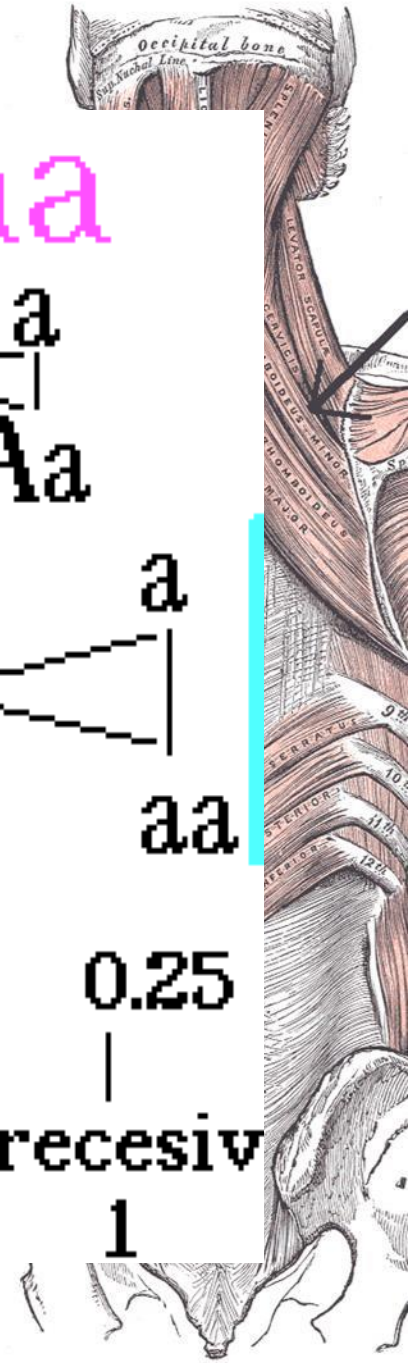
g. a doua gen. AA Aa Aa aa

0.25 0.5 0.25

fenotip a doua generatie

dominant 3

recesiv 1

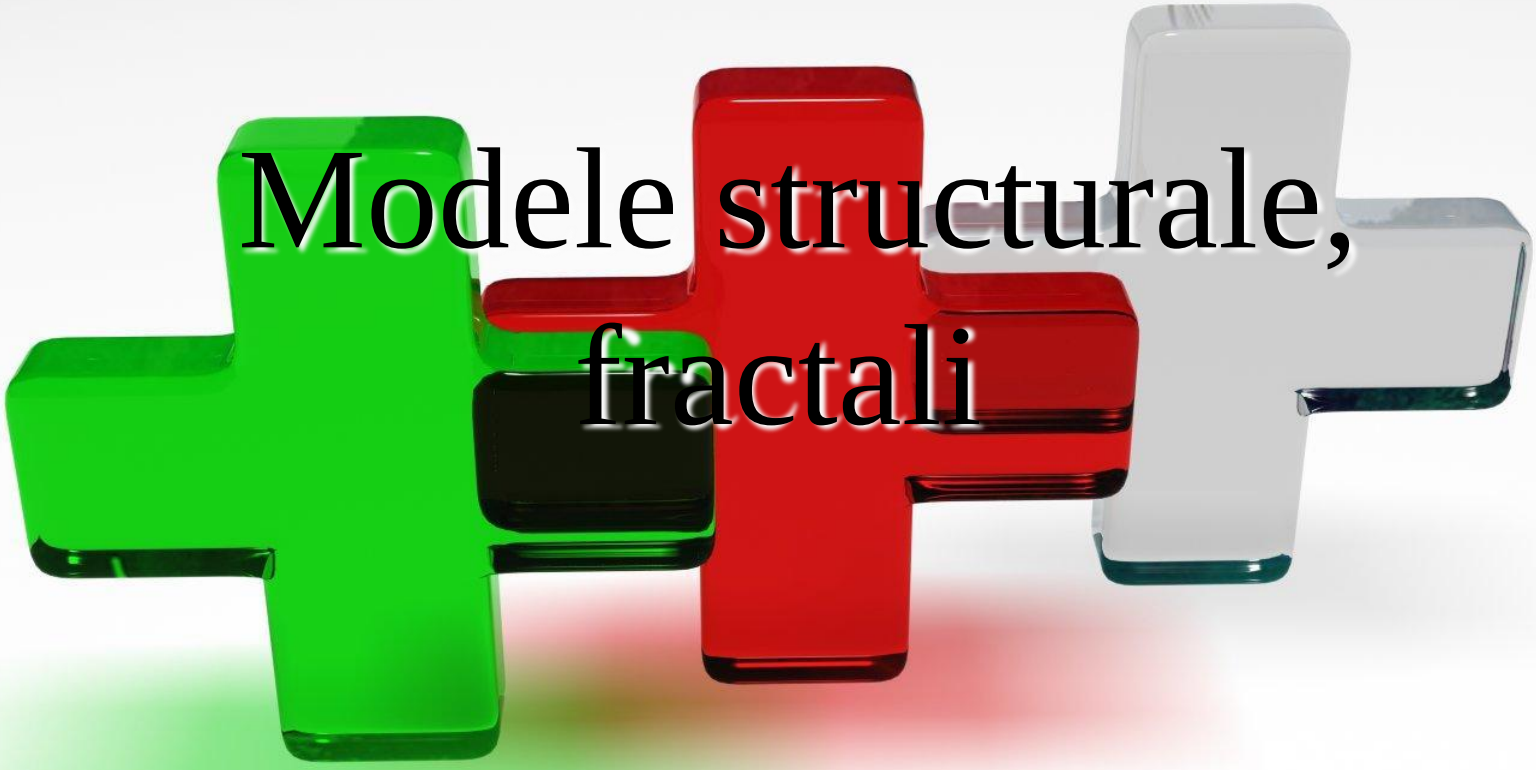


Enunț:

Fie p_A si p_a frecventele a doua alele pe acelasi locus; dupa o generatie de incrucisari intimplatoare, frecventele relative ale urmasilor sunt

p_A^2 , $2p_A p_a$ si p_a^2

Modele structurale, fractali



Simularea prin laturi Markov

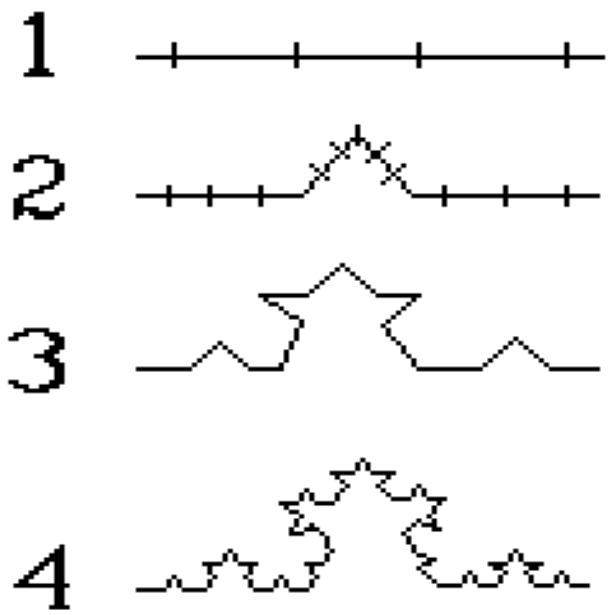
MODELAREA STRUCTURILOR

Transformari morfologice

Sisteme Lindemayer

FRACTALII

Ex: FULGUL LUI KOCH



MODELE DEZVOLTATE

Modelul Lotka-Volterra

a) Sistemul pradator-prada
b) Spatiul fazelor
c) Modele analoage

Modele epidemice

a) Modelul Kermack-McKendric
b) Alte modele

Modele complexe

Modelarea potentialului de actiune al axonului

a) Modelul Hodgkin-Huxley

i. potentialul de repaus

ii. mod. electric al membranei

iii. mecanismul producerii potentialului de actiune

iv. modelarea matematica a declansarii potential.de act.

b) Modelul Fitzhugh-Nagumo

Modelarea complexa in fiziologie -

pachetul "Human"

RETELE NEURONALE

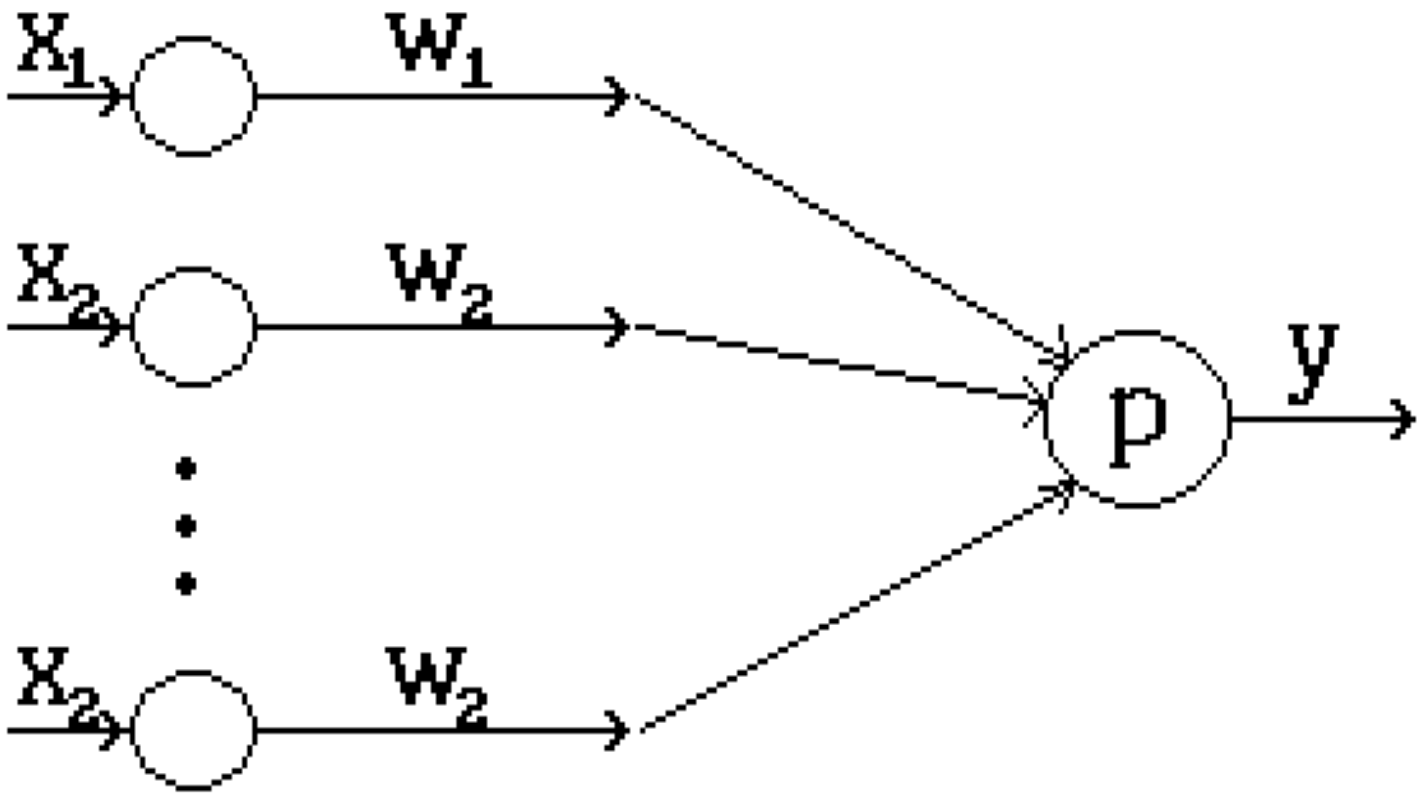
Neuronul formal: Modelul McCollough-Pitts

a) Caracteristicile unui neuron formal

b) Simularea functiilor logice

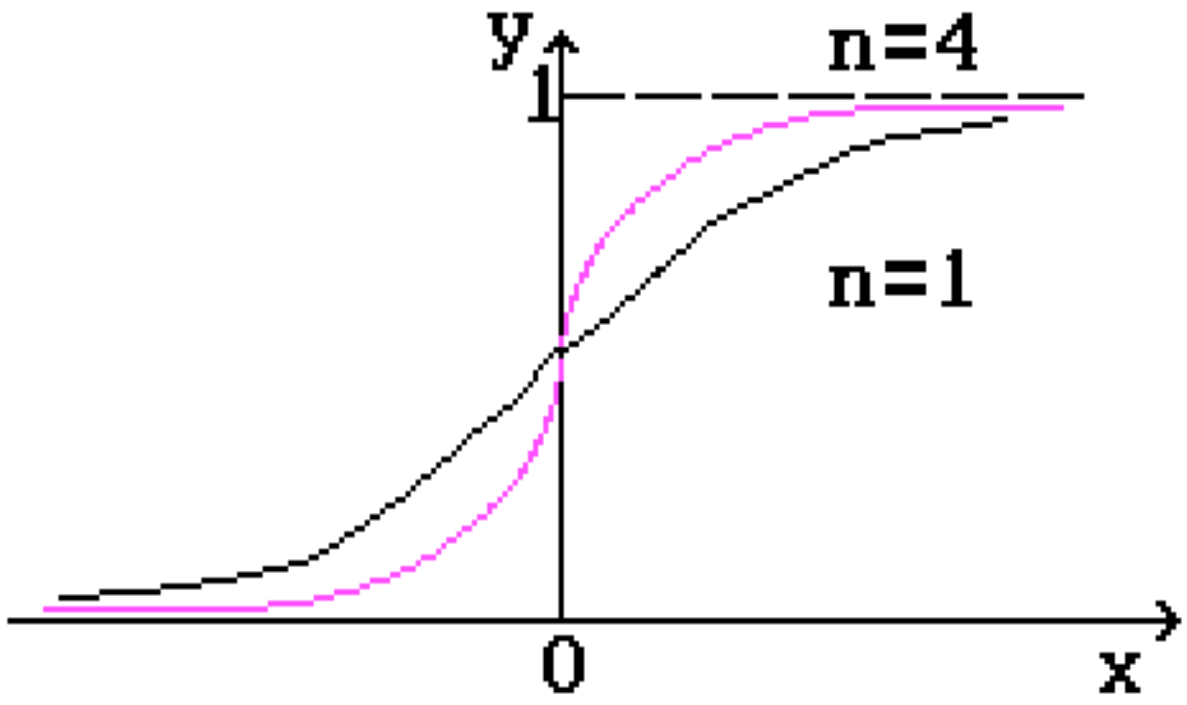
i. ponderi sinaptice

ii. neuronul prag


X1
W1
X2
W2
X2
W2
P
y

Funcția: $$Y = \frac{1}{1 + e^{-nx}}$$

Graficul:



iii. functia-raspuns continua
iv. scheme pentru functii logice

Caracteristicile retelelor neuronale

a) Arhitectura retelei

i. retele nestratificate
ii. retele stratificate

b) Dinamica retelei

ii. retele feed-forward

ii. retele feed-back

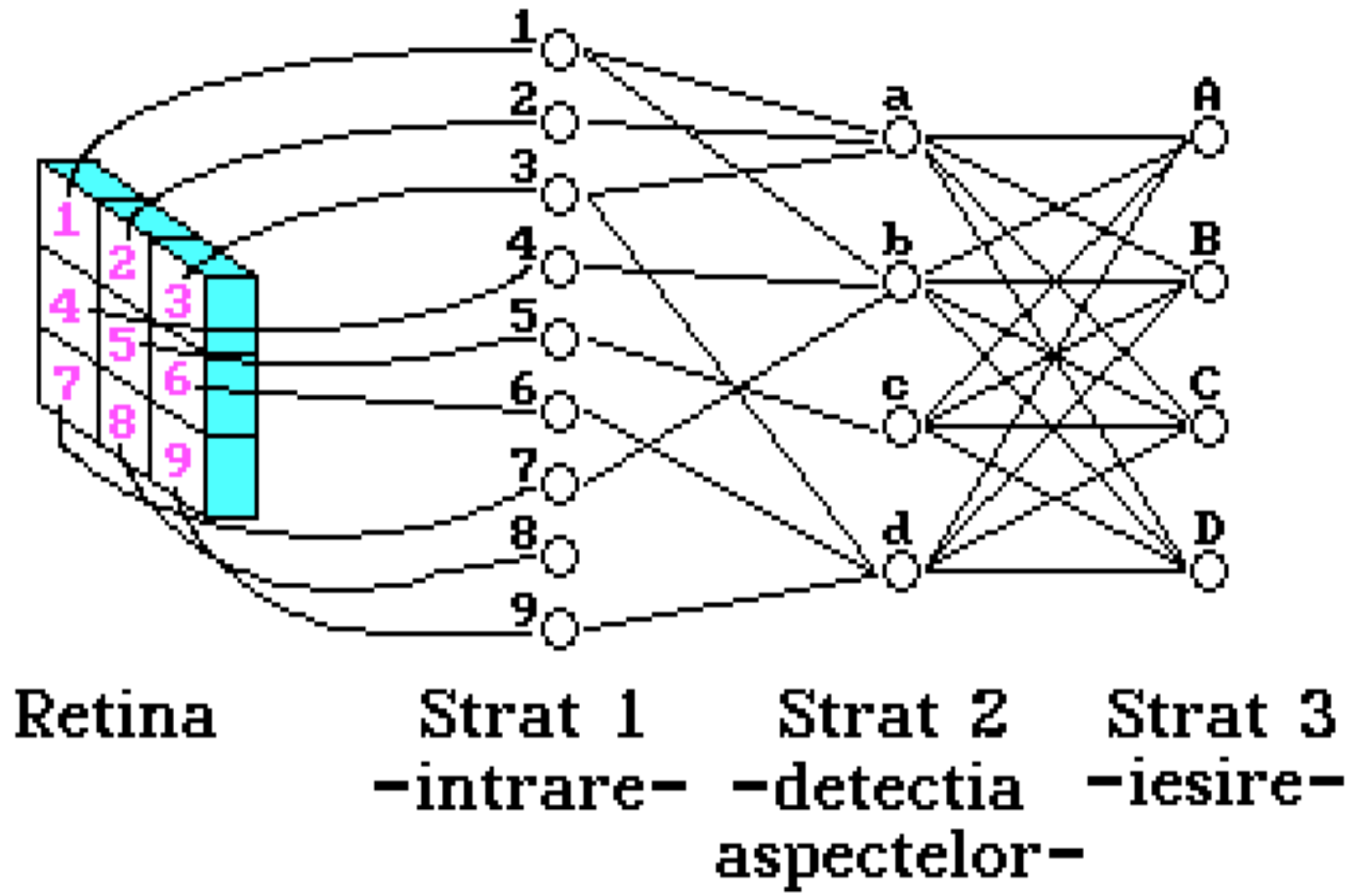
c) Invatarea retelei

i. faza de antrenare

ii. faza de recunoastere

Perceptronul

a) Descrierea perceptronului


1
2
3
4
5
6
7
8
9
a
b
c
d
A
B
C
D
Retina
Strat 1
-intrare-
Strat 2
-detectia
aspectelor-
Strat 3
-iesire-

b) Metode de invatare

i. invatarea supervizata

= regula delta

= backpropagation

= adaline

= madaline

c) Regula lui Hebb

d) Limitarile perceptronului

Retele cu autoorganizare – *Kohonen*

Reteaua autoasociativa – *Hopfield*